(i) **Preamble:** Let \( \{X_n\} \) be a sequence of independent and identically distributed (iid) random variables (rvs) defined over a common probability space \((\Omega, \Phi, P)\) and let \( F \) denote the distribution function (df) of \( X_1 \). Define \( S_n = X_1 + \ldots + X_n \) and \( M_n = \max(X_1, \ldots, X_n) \), \( n > 1 \). If \((S_n)\), properly normalized, converges to a limit law, then it is well known that the limit law is necessarily a stable law. If the limiting stable df is \( G_\alpha \), then \( F \) is said to belong to the domain of attraction of \( G_\alpha \), denoted by \( F \) belongs to \( DA(G_\alpha) \), \( 0 < \alpha \leq 2 \). A few results relevant from the context of our project are iterated logarithm law for \((S_n)\) – Vasudeva (1984); invariance principle for the law of the iterated logarithm – Strassen (1964), functional laws of the iterated logarithm – Pakshirajan and Vasudeva (1980), Wichura (1974); limit theorems for multivariate stable laws – Meerschaert and Scheffler (2001), and so on.

Similarly, when \((M_n)\), properly normalized, converges to a non-degenerate rv, then it is well known that the limit law is either Frechet or Weibull or Gumbel, known in general as max-stable laws. Then the df \( F \) is said to belong to the max domain of attraction of a max-stable law \( H \), denoted by \( F \) belongs to \( max-DA(H) \). Some results of relevance, are laws of the iterated logarithm for \((M_n)\), functional law of the iterated logarithm for \( \{M_{[nt]}\} \), \( 0 \leq t \leq 1 \} – Wichura (1974a), Almost sure limit set of vector of upper order statistics – Vasudeva and Moridani (2011), etc. When \( F \) belongs to \( DA(G) \), where \( G \) is non-normal stable law, one can observe that \( F \) belongs to \( max-DA(H) \). Here, Horvath and Shao (1996), and Lu and Qi (2005) have established law of the iterated logarithm for \(|S_n - M_n|\).

In this project, we extend some of these results to larger classes of distributions, viz., semi-stable laws and max-semi stable laws and their domains of partial attraction.

P.Levy initiated the study of semi-stable laws and observed that they are infinitely divisible and that stable laws are members of this class. He also noted that if over a subsequence \((n(k))\), \((S_{n(k)})\), properly normalized, converges to a limit law, then it is necessarily semi-stable. Here the df \( F \) is said to belong to the domain of partial attraction of the semi-stable law \( G \). This is denoted by \( F \) belongs to \( DPA(G) \). Pillai (1971) and Kruglov (1972) established that the limit laws obtained over \((n(k))\) which are geometrically increasing, are semi-stable and obtained a number of associated results such as density convergence, etc. Divanji and Vasudeva (1989) obtained a law of the iterated logarithm for \((S_n)\) when \( F \) belongs to \( DPA(G) \) when \( G \) is non-normal; Meerschaert and Scheffler (2001) have extended the results of Kruglov (1972) to random vectors.

In the same spirit, Grinevich (1992, 1993) established that over a geometric subsequence \((n(k))\), if \((M_{n(k)})\), properly normalized, converges to a limit law, then such a law is max-semi stable, of which max-stable laws are members. If the limiting max-semi stable law is denoted by \( H \), then \( F \) is said to belong to the max-domain of partial attraction of \( H \), denoted by \( F \) belongs to \( max-DPA(H) \). Grinevich (1993) also obtained the limit laws of \((M_{n(k)})\), under power normalization. Pancheva (2010) has discussed a number of properties of these laws.

Another natural problem of interest would be the study of the joint behaviour of \((S_n, M_n)\), as an extension of Darling (1952), when the underlying df \( F \) belongs to \( DPA(G) \). Equally, the study of trimmed sums, \( S_n - M_n \), has been a topic of interest over the decades.

Semi stable and max-semi stable laws have been used in statistical modelling with heavy-tailed distributions, in the areas of non-life insurance models, stock market models, web transmission models, etc. Huillet (2001) has given a number of applications of semi stable laws.
(ii) Issues of focus:

It is proposed to solve the following problems which are quite challenging.

1. Functional law of the iterated logarithm for \( \{S_{[nt]} \mid 0 \leq t \leq 1\} \), when \( F \) belongs to DPA(G) and functional law for \( (\xi(nt)), 0 \leq t \leq 1 \), where \( \xi(t) \) is a semi stable process.

2. Functional law of the iterated logarithm for \( \{M_{[nt]} \mid 0 \leq t \leq 1\} \), when \( F \) belongs to max-DPA(H).

3. Study of tail behaviour when (i) \( F \) belongs to DPA(G) and (ii) \( F \) belongs to max- DPA(H) and their inter-relations.

4. Studying the large deviation behaviour of trimmed sum, \( S_n - M_n \), \( n > 1 \), and hence establishing
law of the iterated logarithms, assuming that $F$ belongs to DPA(G).

5. Establishing the iterated logarithm laws for the vector of upper order statistics, when $F$ belongs to DPA(H).

6. Obtaining functional law of the iterated logarithm for $((S_{[nt]}, M_{[nt]}), 0 \leq t \leq 1)$, when $F$ has positive support and $F$ belongs to DPA(G).

(iii) Objectives:

Better understanding of the random phenomenon in terms of measures such as sums, averages and extremes. In this direction, the main tool of our study is the law of the iterated logarithm, which produces precise bounds (non-random).

(iv) Reasons and justification:

In repeated occurrences of phenomenon, the study of the asymptotic behaviour plays an important role. When the (weak) limit distributions exist only over subsequences, we are attempting to study the almost sure behaviour of some functions over the whole sequence. As such, our strong limit results are less restrictive than the weak convergence results which hold only over subsequences. The functional laws help in tracing the paths of the phenomenon as a function of time, called the sample paths.

(v) Deliverables:

It is expected that the investigations to be carried out under the proposed project will result in five to six research publications in well known, peer reviewed journals.

(vii) Work plan:

Once the project is sanctioned and the first installment is released, steps will be taken to appoint a JRF following the norms. Simultaneously, study of some research problems will be taken up. It is expected that within the end of first six months, a JRF is appointed and the first technical report is ready. By the end of each year, it is planned to come out with two research articles.

(viii) Program schedule:

It is essentially the same as work plan. Further, during each year steps will be taken to present the results in conferences and seminars and to communicate the papers for publication.

(viii) Proposed budget:

<table>
<thead>
<tr>
<th></th>
<th>I year (in lakh)</th>
<th>II year (in lakh)</th>
<th>III year (in lakh)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Recurring</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HEAD</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>JRF</td>
<td>2.12</td>
<td>2.12</td>
<td>2.36</td>
</tr>
<tr>
<td>Emeritus Scientist</td>
<td>2.40</td>
<td>2.40</td>
<td>2.40</td>
</tr>
<tr>
<td>Travel</td>
<td>0.30</td>
<td>0.30</td>
<td>0.30</td>
</tr>
<tr>
<td>Any other (stationary etc.)</td>
<td>0.15</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td><strong>Non-recurring</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lap top</td>
<td>0.60</td>
<td>---</td>
<td>----</td>
</tr>
<tr>
<td>Equipment</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td>5.57</td>
<td>4.97</td>
<td>5.21</td>
</tr>
</tbody>
</table>

Total budget under the project: Rs.15.75 lakh
(ix) **Outcomes and outputs:**

It is expected that the research work carried out will result in training a candidate towards Ph.D. and in the publication of about six research papers in peer reviewed journals.

(x) **List of publications during the last five years(Listed in the Science index/reviewed in Math. Reviews):**


Publications in the university journal

(x) **Summary of proposed research work:**

In many repetitive phenomena, the partial sum $S_n$ and the partial maxima $M_n$ turn out to be important measures. Large sample approximations of the distributions of these measures may not always be possible, since $(S_n)$ and $(M_n)$, normalized, may not converge to limit distributions. However, such convergence may hold only over subsequences. In such situations, we obtain almost sure mathematical bounds for the growth of these measures or examine their sample paths, in terms of the iterates of the logarithms, over the whole sequence. We also aim at obtaining large deviation probabilities of these measures.